Measurement 152 (2020) 107274

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

Methods to calibrate a three-sphere scale bar for laser scanner performance evaluation per the ASTM E3125-17

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ARTICLE INFO

Article history:
Received 27 August 2019
Received in revised form 8 November 2019
Accepted 14 November 2019
Available online 18 November 2019

Keywords:
ASTM E3125-17
Three sphere scale bar
Laser tracker
Terrestrial laser scanner
Length calibration

ABSTRACT

A scale bar with sphere targets is one way of realizing the symmetric and asymmetric length tests to evaluate Terrestrial Laser Scanner (TLS) performance per the ASTM E3125-17 standard. The length of the scale bar is required to be known with an expanded ($k = 2$) uncertainty that must be at least four times smaller than the manufacturer's maximum permissible error (MPE) specification of the instrument being tested. In this paper, we propose two methods to calibrate the scale bar length using a laser tracker. The first method, which we refer to as the four-orientation and two-face (FOTF) method, is proposed for calibrating the scale bar when it is in a horizontal orientation. The other method, which we refer to as the comparison method, is proposed for calibrating the length of the scale bar in any orientation. We describe the methods, present underlying theories, discuss validation experiments, and summarize results. The two calibration methods are beneficial for the realization of the ASTM E3125-17 standard for TLS performance evaluation.

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1. Introduction

In recent years, spherical coordinate three-dimensional (3D) imaging systems such as Terrestrial Laser Scanners (TLSs) are being used more widely in different fields, such as historical preservation and archiving, reverse engineering, geographic modeling, dimensional metrology and assembly of large structures. TLS measurement performance is a key concern for users, especially because factors such as vibration during use, installation and transport of the scanner, etc., can degrade its performance over time. Over the years, researchers and engineers have developed some methods and schemes for performance evaluation of TLSs. This is a wide ranging topic with considerable literature. We, therefore, restrict our discussion to literature pertaining to the development of documentary standards and scale bars for the performance evaluation of TLSs.

The first comprehensive scanner performance study was reported by Boehler in 2003 [1], where the authors recognized the lack of standardization in the specifications provided by TLS manufacturers. Phillips et al. [2] and Beraldin [3] discuss the challenges and recent developments in the area of standards development for laser scanners. Recognizing the need to standardize the performance evaluation of TLS, the ASTM E57 committee on 3D Imaging Systems established a working group in 2006 to develop a documentary standard. The working group recognized that developing test procedures for all the influence factors will be enormously challenging and therefore limited their scope to the evaluation of relative range errors only. See references [4–6] for selected publications from members of that working group. That working group released the ASTM E2938-15 [7] in 2015 for specifying and evaluating the relative range performance of 3D imaging systems. In 2013, another ASTM working group was formed to build on the previous effort by developing a standard to evaluate the point-to-point distance performance of TLS anywhere in the work volume, not simply along the ranging direction. See references [8–10] for selected publications from members of that working group. That working group released the ASTM E3125-17 [11] in 2017. Within ISO, there are currently published field check standards for surveying instruments detailed in the ISO 17123 series [12]. While there are no published standards for TLS systems within ISO, there is ongoing work within ISO TC172 to develop a field check procedure for TLS systems [13].

Research into the development of long scale bars (>1 m) for dimensional measurement system evaluation is well documented in the literature, particularly for Cartesian coordinate measuring machines (CMMs) and laser trackers. These scale bars typically have either sphere targets at the ends or allow for the mounting

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https://doi.org/10.1016/j.measurement.2019.107274
0263-2241/Published by Elsevier Ltd.

of spherically mounted *retro*-reflectors (SMRs) at the ends. Hudlemeyer et al. [14] described the design and evaluation of a prototype scale bar for field testing of laser trackers, and developed a method for in-situ field calibration of a scale bar using a laser tracker. ISO 15530-3:2011 [15] describes a method for the evaluation of measurement uncertainty on a CMM using calibrated workpieces or measurement standards. The technique is closely related to one of the methods, the comparison method, that we discuss in this paper. However, due to its limited working volume, a CMM is not a good choice for calibrating the three-sphere scale bar in different orientations (horizontal, diagonal, or vertical). Further, a CMM cannot be used to calibrate the scale bar in the place where laser scanner performance is evaluated per the ASTM E3125-17. Long scale bars suffer change in length due to gravitational effects and the method of mounting as described in Sawyer et al. [16]. In the existing literature, no convenient methods are presented to calibrate the length of a three-sphere scale bar, which is one of the ways to realize TLS performance evaluation per the ASTM E3125-17 standard. In this paper, we describe a novel three-sphere scale bar and present two methods for calibrating this scale bar length using a laser tracker, the four-orientation and two-face (FOTF) method and the comparison method. We point out that the FOTF method is the novel research contribution of this paper. The comparison method is widely used in metrology; here we show a practical application for scale bar calibration using laser trackers.

The paper is organized as follows. We describe the three-sphere scale bar in Section 2, the line-of-sight (LOS) method to obtain reference values in Section 3, our proposed FOTF method in Section 4, our proposed comparison method in Section 5, uncertainty analysis in Section 6, and conclusions in Section 7.

2. The three-sphere scale bar

As mentioned earlier, a scale bar with three spheres is one way of realizing the fourteen symmetric and asymmetric length tests described in the ASTM E3125-17 standard. Different views of the three-sphere scale bar are shown in Fig. 1. The bar is about 2.3 m long and made of carbon fiber tube with a rectangular cross section. Three specialized aluminum spheres with a nominal diameter of 10 cm are mounted at each end and at the middle of the bar. The surfaces of the spheres have a matte gray finish providing diffuse reflection that is scanner friendly. Each sphere is hollow, with a kinematic nest located inside that allows a 38.1 mm (1.5 in) SMR to be centrally placed as shown in Fig. 1(b). By design, the sphere center is coincident with the center of the inner SMR (See description of 'Sphere concentricity with SMR' and 'SMR concentricity' at the start of Section 6 for details). In order to facilitate TLS scanning and reduce back-scattering, the scale bar surface is covered with a layer of black laser absorbing fabric over its surface except in the region containing the three spheres.

In an early prototype of the bar shown by all the sub-figures of Fig. 1, each sphere has two small holes drilled in such a manner that the three centers and the six holes of the three spheres are collinear when the spheres are mounted on the bar. These holes are intended to allow the calibration of the sphere center-to-center distance by aligning a laser beam of a laser tracker along the length of the bar. The diameters of these holes are about 5 mm, these are larger than the laser beam diameter employed by the laser tracker. While this idea of aligning the laser beam of a laser tracker through small holes allows for the calibration of the bar in any orientation (horizontal, diagonal, or vertical), it is a time consuming and labor-intensive process. In this paper, when measuring the distances between two or more targets that are nominally collinear with the laser tracker (the difference in azimuth and elevation angles of the targets are typically smaller than 0.05° in this study), the measurement method of aligning a laser beam of a laser tracker along the line joining the targets directly or through a mirror is referred to as the line-of-sight (LOS) calibration, see Fig. 2. For the three-sphere scale bar, this LOS calibration method requires considerable care in the manufacture of the bar to ensure that the six small holes and the sphere centers are collinear. To overcome these challenges, we propose alternate calibration methods in this paper that do not require the use of these six holes, thus greatly reducing the manufacturing burden on the bar, and simplifying the calibration process. The LOS method is commonly used to obtain high accuracy length measurements using laser trackers. We use this method to validate our two proposed methods and briefly describe it in the next section.

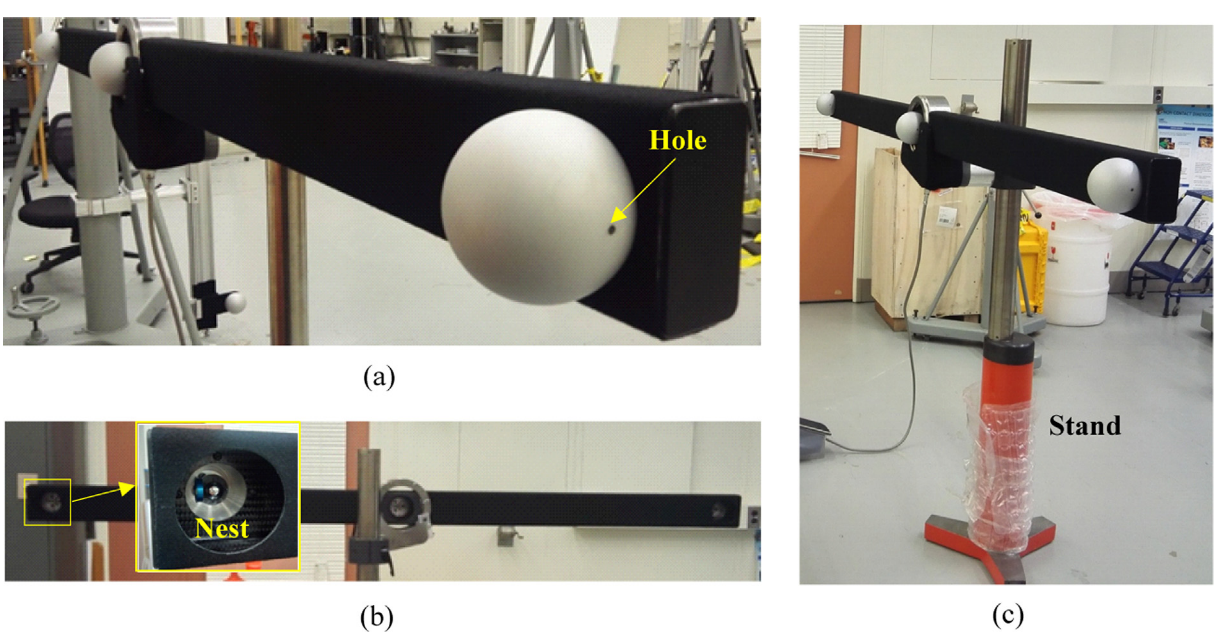

Fig. 1. Different views of the three-sphere scale bar, (a) showing a sphere at the end of the bar and a hole on the side for the laser beam, (b) showing that each sphere is hollow with a kinematic nest located inside, (c) showing the full view of the three-sphere scale bar and its stand.

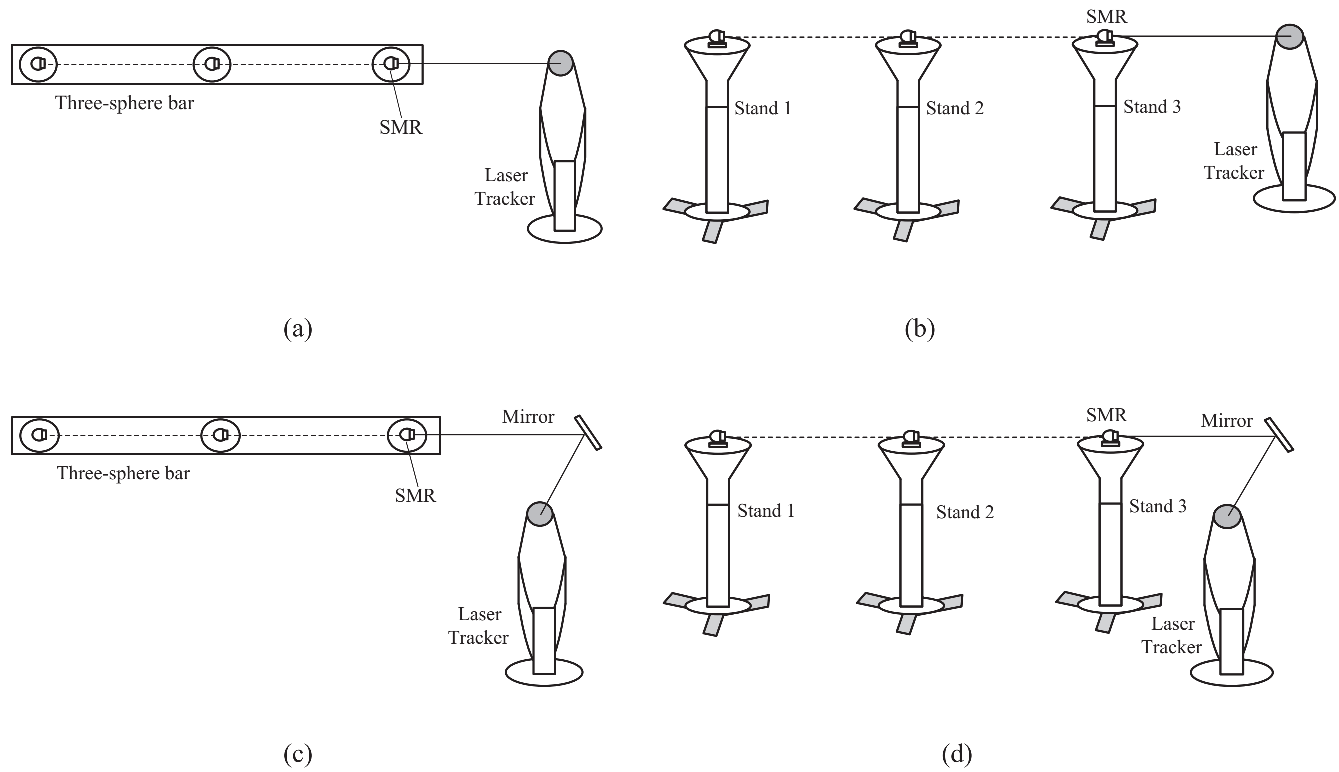

Fig. 2. Line-of-sight (LOS) calibration by physically aligning the laser tracker to be collinear with the nests for the case of (a) three-sphere bar, and (b) three stands. Line-of-sight (LOS) calibration using a mirror to align the laser beam of a laser tracker to be collinear with the nests for the case of (c) three-sphere bar, and (d) three stands.

3. Line-of-sight (LOS) method

In the LOS method, we align the laser beam of the laser tracker along the line joining the nests. In such a case, the laser tracker can obtain the coordinates by placing the SMR at each nest successively. The layouts of the LOS calibration for the lengths of three-sphere bar and the distances between three stands are shown in Fig. 2. Note that while the primary objective of this paper is to use the LOS method to verify our other proposed methods for the calibration of the three-sphere scale bar, we also use the LOS method on stands (instead of on the three-sphere scale bar) to validate our proposed methods.

In order to minimize the errors introduced due to the angular measurement of the laser tracker, we ensure that the differences among the measured horizontal/vertical angles of the nests are typically less than 0.05°. This process is accomplished through manual adjustment/alignment of the laser tracker. Thus, the measurement error in the length doesn't include the effects of any of the misalignment geometric errors described in Section 4.2. It is only the ranging errors of the laser tracker that influence the length measurements. Furthermore, the LOS method can be used for measuring the length of the scale bar at vertical or other angular orientations if we can ensure the origin of the laser tracker and the measured points are collinear. The alignment process can be performed by physically moving the tracker so that its laser beam is along the line joining targets. However, it is more convenient to place a mirror near one of the nests and adjust the tilt of the mirror so that when the laser bounces off the mirror, it is in-line with the two or three nests. We discuss the uncertainty in the LOS method in Section 5. In this paper, we use this method to validate both the FOTF method and the comparison method.

We also note that we use the absolute distance mode (ADM) of the laser tracker instead of the interferometer mode (IFM) because it is more convenient to use. We have checked the ADM against our reference interferometer and determined that it meets the stated manufacturer's specification of 10 μm error through out its range.

4. Method to calibrate the scale bar in the horizontal orientation

4.1. Four-orientation and two-face (FOTF) method of measuring scale bar length

This method involves placing the laser tracker centrally behind the scale bar, at the same height as the scale bar, and at a distance that allows the laser beam to enter each SMR without obstruction (see Fig. 3). With the laser tracker in this orientation, each SMR is measured in both front- and back-face. The front-face of a laser tracker may be arbitrarily defined based on the zenith angle of the laser beam directed to a target. For example, if the laser beam emerges when the zenith angle is between 0° and 180°, that face of the instrument may be noted as the front-face. Otherwise, when the zenith angle is between 180° and 360°, that face of the instrument is noted as the back-face. The average from the front-face and back-face coordinates is used to determine the sphere center-to-center distances. The laser tracker is then manually rotated by 90°±5° in its azimuthal plane and the sphere centers are measured again. This process is repeated two more times so that the sphere center-to-center distances are obtained for four orientations of the laser tracker, as shown in Fig. 3 (the three-sphere scale bar is supported at the middle). These distances are then averaged to obtain the final sphere center-to-center distances. We will demonstrate that this method yields sphere center-to-center distances that are within ± 5 μm of the length obtained by aligning a laser beam along the length of the bar, i.e., through the six small holes on the sides of the spheres and striking the SMRs in the nests.

We note that the FOTF method not only works with the specialized spheres (i..e, spheres with integrated nests), but also with

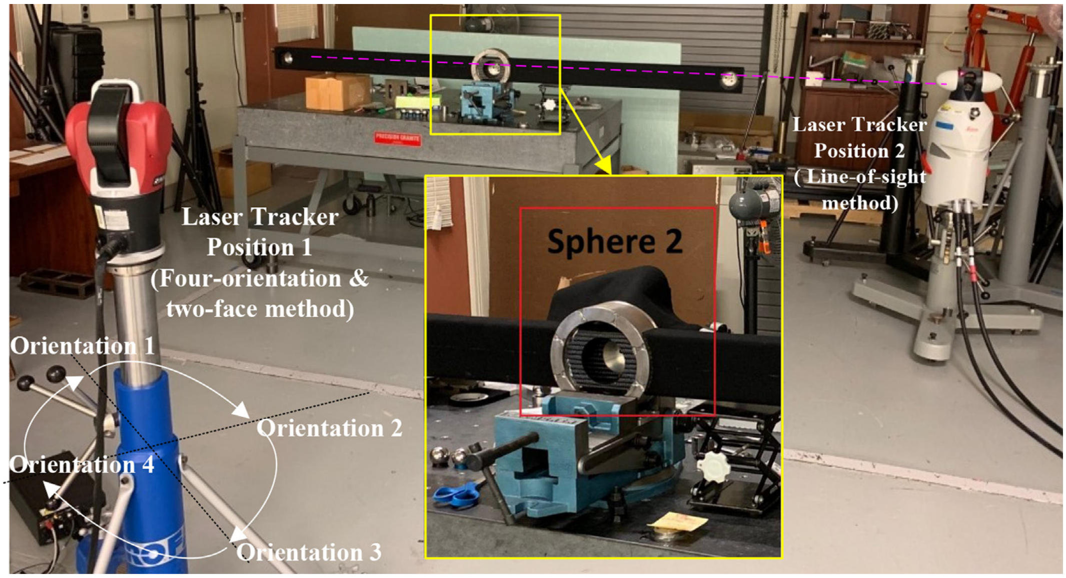

Fig. 3. Layouts of four-orientation and two-face method and line-of-sight method.

solid spheres mounted on a scale bar. In that case, the laser tracker has to be placed in front of the scale bar and several points have to be manually acquired on the surface of each sphere to determine their centers.

4.2. Model based explanation

Laser trackers are spherical coordinate measurement instruments, and they are assembled with several components that may contain misalignments within their construction. These misalignments, which are typically geometric offsets, tilts and eccentricities, result in systematic errors in the measured point coordinates. Muralikrishnan et al. [17] developed a model for laser trackers without a beam steering mirror, given as follows:

$$Rc = Rm + x_2 sin(Vm) + x_8 \tag{1}$$

$$Hc = Hm + \frac{x_{1t}}{Rm\ \sin(Vm)} + \frac{x_{4t}}{\sin(Vm)} + \frac{x_5}{\tan(Vm)} + x_{6x}\cos(Hm) - x_{6y}\sin(Hm) + x_{9a}\sin(2\ Hm) + x_{9b}\cos(2\ Hm) \tag{2}$$

$$Vc = Vm - \frac{x_{1m}}{Rm} + \frac{x_2\cos(Vm)}{Rm} + x_3 + x_{7n}\cos(Vm) - x_{7z}\sin(Vm) + x_{10a}\sin(2\ Vm) + x_{10b}\cos(2\ Vm) \tag{3}$$

where:

Rc, Hc, and Vc are the corrected range, horizontal angle, and vertical angle;
Rm, Hm, and Vm are the measured range, horizontal angle, and vertical angle by the laser tracker;
x_{1t} and x_{1m} are the beam offset components along the transit and the transit normal, respectively;
x_2 is the transit offset;
x_3 is the vertical index offset;
x_{4t} is the beam tilt components along the transit;
x_5 is the transit tilt;
x_{6x} and x_{6y} are the horizontal angle encoder eccentricity resolved into X and Y components;
x_{7n} and x_{7z} are the vertical angle encoder eccentricity resolved into N and Z components;
x_8 is the bird bath error;
x_{9a} and x_{9b} are the second order harmonic scale errors in the horizontal angle encoder;
x_{10a} and x_{10b} are the second order harmonic scale errors in the vertical angle encoder.

A detailed description of the errors above are presented in reference [17]. Based on the model in Eqs. (1)–(3), the relationship between the sensitivity for each misalignment parameter and the laser tracker performance tests was obtained and discussed in [17].

We now discuss how this laser tracker model allows for the calibration of the scale bar length with low uncertainty by the following method. As discussed in Section 4.1, we measure the length of a horizontal scale bar at four orientations of the laser tracker (0°, 90°, 180°, and 270°) in both front-face and back-face modes. The scale bar is at approximately laser tracker height and centrally placed with respect to the laser tracker (The three nests are approximately 4 m, 3.8 m, and 4 m away from the laser tracker). The final reported sphere center-to-center distance is the average of the four measurements from the four orientations of the laser tracker. Hence, we refer to the method as the four-orientation and two-face (FOTF) method.

As discussed in reference [17], ten of the above 16 parameters (x_{1t}, x_{1m}, x_2, x_3, etc.) are sensitive to two-face measurements, i.e., the signs of the errors reverse between front-face and back-face measurements. Therefore, averaging front-face and back-face measurements eliminates the effects of these parameters. The only terms that remain are x_8, x_{7z}, x_{9a}, x_{9b}, x_{10a}, and x_{10b}.

Because the measured vertical angle Vm is close to 90° in the FOTF method, the contribution of term x_{10a} is zero considering $\sin(2Vm) = 0$. Because we measure from four orientations and average the results of these orientations, the contributions of the terms x_{9a} and x_{9b} drop to zero (because $\sin(2Hm)$ and $\cos(2Hm)$ are periodic functions). The only remaining terms are x_{7z}, x_8 and x_{10b}. Because of the geometry of the setup, we can tolerate a large error due to x_{7z} and x_{10b} but a ranging error x_8 will have a direct impact on the measured length (see Fig. 4) that is proportional to the sine of the angle θ (where θ is the angle between the laser beam OC and the perpendicular OB from the laser tracker to the horizontal scale bar, see Fig. 4). In order to reduce the effect of the term x_8, we move the laser tracker farther away from the bar. As shown in Fig. 4, e_{Hm} is the swing error of the measurement of point C and is perpendic-

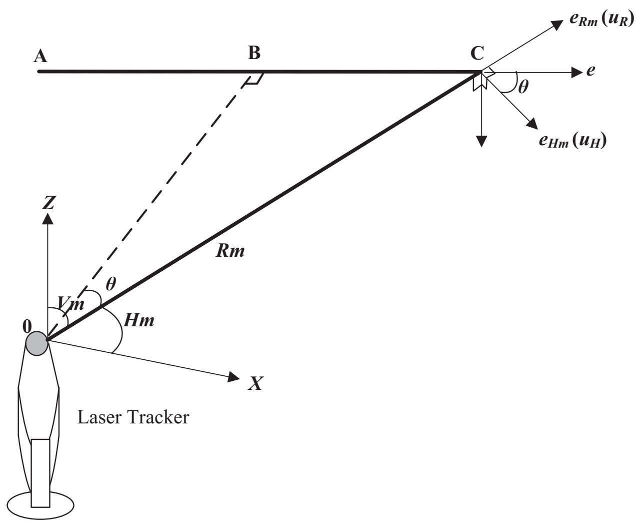

Fig. 4. Relationship between measurement geometry and errors.

ular to the laser beam *OC*, which is caused by the measurement error of the angle *Hm*. Let e_{Rm} be the ranging error. Then, the error incurred at *C* along the length *AC* is $e_{Rm} \times \sin(\theta) + e_{Hm} \times \cos(\theta)$. In such a case, the larger the measured range *Rm*, the smaller the angle θ due to the constant bar length (see Fig. 4), and the therefore, the smaller the effect of an error in *Rm* on the length of the scale bar.

As analyzed above, when measuring the length of the scale bar set horizontally and placed at the laser tracker height in the FOTF method, the systematic errors from the laser tracker presented in Eqs. (1) to (3) can be almost eliminated. Thus, the measurement error of the scale bar length is small and on the order of the errors obtained when calibrating the sale bar using the LOS method.

4.3. Validation experiment for four-orientation and two-face method

There are two parts to the validation experiments we performed. First, we used a setup with three stands and nests instead of the three-sphere scale bar and compared the FTOF and LOS methods. This was done to isolate any issues pertaining to the design or manufacture of the bar that may contribute to measurement errors. After that experiment was performed, we then compared the measurements on the three-sphere scale bar using both the FOTF and LOS methods. We describe these experiments and results here.

In order to validate the FOTF method, we placed a nest on each of three stands so that the nests were at the same height from the ground and collinear, as shown in Fig. 5. The distance between two adjacent nests was about 1.15 m. The motivation of this validation experiment is to demonstrate that the FOTF method yields center-to-center distances that were within a few micrometers of that obtained from the LOS method. In Fig. 5, one laser tracker is located in Position 2 for the LOS method, and the other laser tracker is located in Position 1 for the FOTF method.

In this subsection, we describe the validation experiment of the FOTF method by comparing it against the LOS method. Both the LOS and FOTF measurements were repeated ten times. Before calculating the length standard deviations in the FOTF method, coordinate measurements of an SMR in a nest from both faces of the laser tracker are first averaged. Then, the forty measured lengths in the four orientations of the laser tracker are used for calculating the standard deviation. The result of the experiment is shown in Table 1.

As shown in Table 1, although the standard deviations of the measured lengths for the FOTF method are slightly larger than that of the LOS method, the mean lengths of the two methods are within 5 μm of each other. This result indicates that the systematic error of the laser tracker has been removed when averaging the horizontal scale bar lengths of the four laser tracker orientations in the FOTF method. To verify this conclusion for the specific case of the three-sphere scale bar, we repeat the same experiment on the scale bar.

As mentioned in Section 2, each sphere of the three-sphere scale bar has two holes, and the six holes and the three sphere centers are collinear (i.e., the laser beam of the laser tracker passing through the six holes can pass through the centers of the three spheres.). Therefore, the laser beam of a laser tracker can be aligned through the six holes when the scale bar is horizontal, as shown in Fig. 3. If we place the SMR in the nest inside each sphere, then the distances between the sphere centers can be measured by the LOS method using the laser tracker at the position 2 in Fig. 3. After the LOS measurement, we measure the scale bar using the FOTF method with the laser tracker at position 1 in Fig. 3.

In the FOTF method, 10 repeated measurements are performed for each orientation of the laser tracker. In the LOS method, 10 repeated measurements are performed for each sphere center when the laser tracker is in a line with the centers of the three spheres. The three-sphere scale bar is supported at the middle. The uncertainties of these results are given in Sections 6.1 and 6.2.

The measurement results for this case are also shown in Table 1. As shown in Table 1, the mean lengths of the three-sphere scale bar by the two methods differ by less than 5 μm, which is similar with that of the three-stand case. This result also shows that the systematic errors of the laser tracker as given in Eqs. (1) to (3) can be removed by the FOTF method, for the case of the horizontal scale bar length measurement. Additionally, in Table 1, the length L23 is close to the difference of the lengths L13 and L12. Because of possible non-collinearity of the three sphere centers, we calculate and report all three lengths in the table.

The experiments above show that the FOTF method provides comparable results to the LOS method, and it can be used in case of inconvenience or inability to use the LOS method for a horizontal scale bar length calibration. We note that the experimental one sigma number in Table 1 hides that fact that the length actually changes in a systematic manner from one orientation to the next because of systematic errors in the tracker. In the FOTF method, we remove this systematic error by taking the average from the four orientations and therefore obtain a length that is close to the LOS length. We discuss the uncertainty in the LOS and FOTF methods in Section 6.

5. Method to calibrate the scale bar in any orientation

Long length scale bars change in length based on mounting and orientation as shown by Sawyer et al [16]. While the previously described method allows for the calibration of the scale bar length in the horizontal orientation, there is a need to determine the length of the scale bar in other orientations to quantify the change in length between orientations. We discuss this here.

5.1. Comparison method of measuring scale bar length

The FOTF method is effective for calibrating the scale bar in the horizontal orientation, as verified in the last section. However, when the scale bar is in vertical or in a diagonal orientation, the scale bar length cannot be calibrated by the FOTF method because the systematic errors in the laser tracker are not removed by averaging from four orientations. For the case

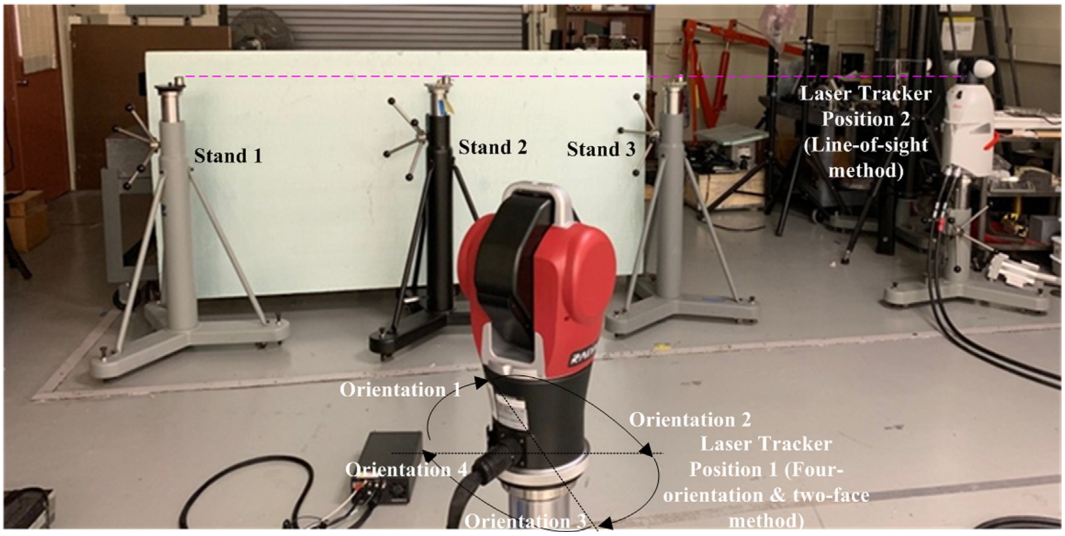

Fig. 5. Three-stand horizontal length measurement layout.

Table 1
Three-stand/three-sphere horizontal scale bar length by the LOS and the FOTF methods.

	Length name	FOTF method (mm)		LOS method (mm)		Difference in the mean length from the two methods (mm)
		Mean	Standard deviation	Mean	Standard deviation	
Three-stand setup	L12	1150.010	0.005	1150.009	0.001	0.001
	L13	2299.983	0.004	2299.980	0.001	0.003
	L23	1149.974	0.006	1149.971	0.001	0.003
Three-sphere scale bar	L12	1149.751	0.007	1149.750	0.002	0.001
	L13	2299.627	0.009	2299.625	0.002	0.002
	L23	1149.878	0.006	1149.876	0.002	0.002

*L*ij* is the length between stand (sphere) *i* and stand (sphere) *j*, and *i*, $j \in \{1,2,3\}$.

of the vertical or diagonal orientations, it is rather cumbersome to align a laser beam through the six small holes on the sides of the sphere to perform an LOS calibration. Further, as pointed out in Section 2, if the manufacturer decides to produce the scale bar without the side holes, it is necessary to establish another calibration method to determine the sphere center-to-center distances for vertical and diagonal orientations. The calibration of the length in different orientations is necessary to establish that the length of the scale bar has not changed significantly due to self-weight. In this section, we detail the calibration method of the bar length, called the comparison method. We note that the comparison method has been used extensively in dimensional metrology. The novelty here lies in the application of this method to calibrate scale bars using laser trackers, which to our knowledge, has not been done before.

The main idea of the comparison method is that the systematic errors of a laser tracker in Eqs. (1) to (3) are the same when the measured points have the same ranges, horizontal and vertical angles. Therefore, if two scale bars have similar lengths, and if one of the two scale bars can be calibrated by the LOS method, then it can be used as a master artifact to calibrate the other scale bar. Unlike the FOTF method, we can calibrate the scale bar in any orientation – horizontal, vertical, or diagonal.

For the purposes of this discussion, we only consider a scale bar with spheres targets at its ends, although the same method can be also applied to a three-sphere scale bar.

The procedure of the comparison method is as follows (see Fig. 6):

(1) Suppose we have two scale bars, which we refer to as the master bar and the test bar. It is assumed that there exist two SMR nests, one at each end of the master bar, located nominally the same distance apart from each other as in the test bar. In our case, the three-sphere scale bar is the test bar for which we desire the length between the two extreme spheres in different orientations.
(2) We set up the test bar at the desired orientation and measure the length of the bar using a laser tracker placed in front of the bar at an arbitrary location (see Fig. 6(a)). We refer to this as the direct measurement. It is not necessary that the SMR inside the spheres be measured on both faces, but we do so anyway. Clearly, the errors inherent in the laser tracker manifest as errors in the SMR location and therefore in the sphere center-to-center distances of the test bar. Let the length so obtained be referred to as $L_{\text{test,direct}}$.
(3) We then place the laser tracker next to the master bar in such a manner that the range, horizontal and vertical angles to the two nests are identical (the differences of the angles are typically less than 0.1°) to that of the test bar as shown in Fig. 6(b). Clearly, to achieve this, the master bar must be in the same relative orientation as the test bar and the laser tracker must be placed at the same location relative to the master bar as it was placed relative to the test bar. We measure the nests, let the length so obtained be $L_{\text{master,direct}}$.
(4) Using a mirror, we then align the laser beam of the laser tracker to perform an LOS measurement of the master bar. Let the length so obtained be $L_{\text{master,LOS}}$ as shown in Fig. 6(c).

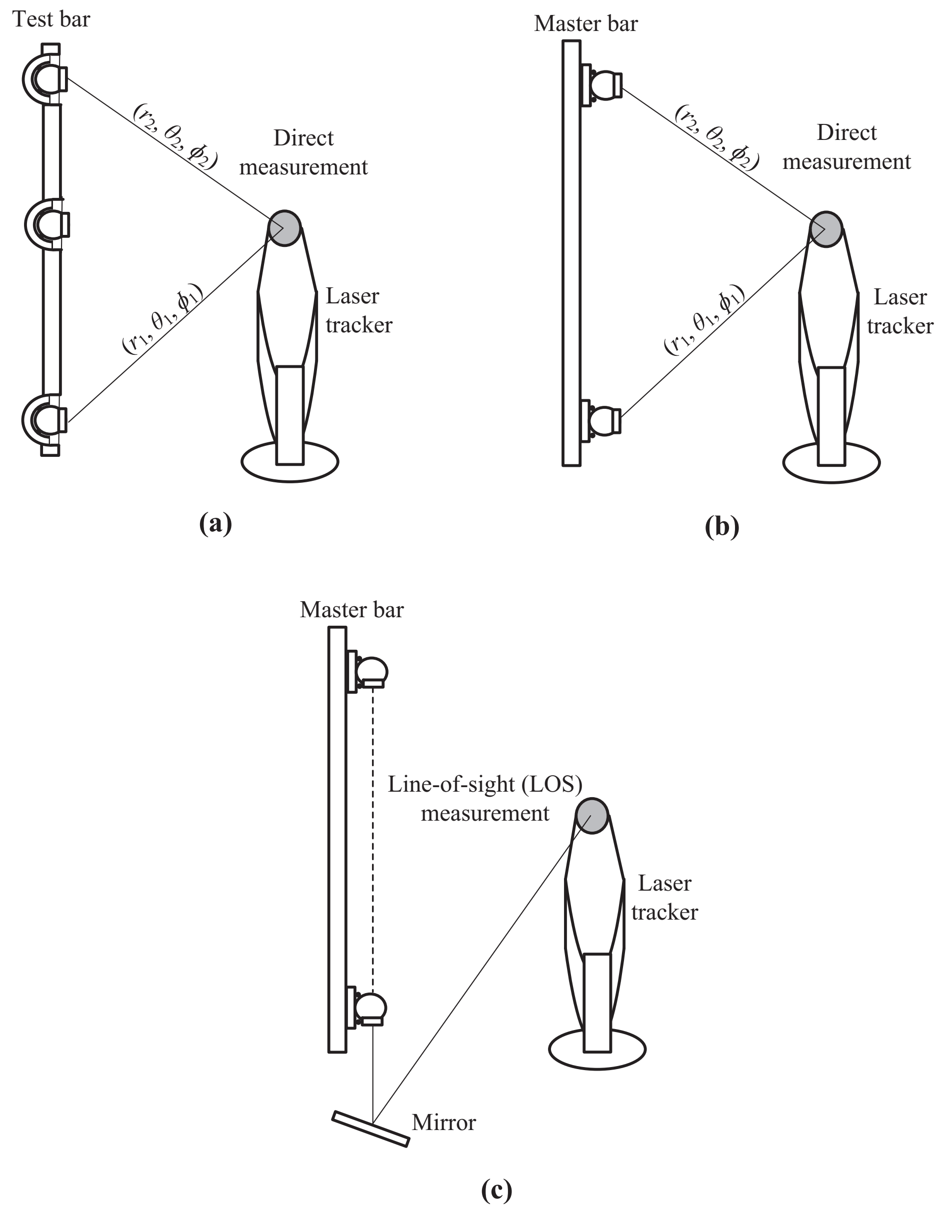

Fig. 6. Measurement modes of comparison method when the scale bars are vertical, (a) showing step 2 where the test bar is measured by the laser tracker in the direct mode, (b) showing step 3 where the master bar is measured in direct mode with care taken to ensure that the range and angles to the two ends of the scale bar are as close as possible to the measurement in step 2, (c) showing step 4 where the master bar is measured in the LOS method.

(5) The error in the length of the master bar by the direct measurement method is $e = L_{\text{master,direct}} - L_{\text{master,LOS}}$. This same error manifests in the direct measurement of the test bar because the coordinates of the nests are identical for both the test and master bar measurements.

(6) Therefore, the calibrated length of the test bar is obtained by correcting the measured length with error e. Thus, $L_{\text{test,LOS}} = L_{\text{test,direct}} - e$.

5.2. Validation experiment for comparison method

In order to verify the proposed comparison method, a validation experiment is performed using two scale bars both of which allow line-of-sight measurements. That is, the test bar has 38.1 mm (1.5 in) SMR nests instead of sphere targets. We refer to one as the master bar and the other as the test bar. The test bar is setup at a desired orientation and measured in the direct mode. The master bar is then measured in the direct mode with care taken to ensure that the horizontal and vertical angles to the two ends are as close as possible to those on the test bar (the differences of the horizontal and vertical angles are suggested to be less than 0.1°). The master bar is subsequently measured in the line-of-sight method using a mirror. The error in the master bar length due to systematic errors in the laser tracker is calculated, and this value is used to correct the direct method measurement of the test bar. Finally, the test bar is measured in the LOS method to establish that the previously obtained value for the corrected direct method value is within a few micrometers of the LOS measurement.

We performed these validation experiments for the case of the horizontal and vertical orientations. Because the length error of a diagonally oriented scale bar is likely between that obtained in the horizontal and vertical orientations, the case of the diagonal orientation is not included in this experiment.

Table 2
Horizontal and vertical lengths of two bars by the comparison method.

Bar posture	Bar name	Line-of-sight measurement (mm)		Direct measurement (mm)		Systematic error in master bar length due to laser tracker error (mm)	Corrected length of the test bar in the comparison method (mm)	Error in the corrected length of the test bar (mm)
		Mean	Std	Mean	Std			
Horizontal	Master	2300.049	0.002	2300.055	0.003	2300.055–2300.049 = 0.006	/	/
	Test	2339.500	0.002	2339.505	0.004	/	2339.505–0.006 = 2339.499	2339.499–2339.500 = –0.001
Vertical	Master	2300.054	0.002	2300.056	0.002	2300.056–2300.054 = 0.002	/	/
	Test	2339.498	0.002	2339.497	0.004	/	2339.497–0.002 = 2339.495	2339.495–2339.498 = –0.003

* The uncertainties of these results are given in Section 6.

The results are shown in Table 2. Ten repeated measurements were performed for both the LOS and the direct measurements. In this experiment, the differences between the horizontal/vertical angles of the ends of the two scale bars were kept to less than one degree when their lengths are measured by the laser tracker in the direct mode.

It can be seen in Table 2 that the comparison method results are 2339.499 mm and 2339.495 mm for the horizontal and vertical orientations of the test bar, respectively. These values are within 5 μm of the corresponding line-of-sight measurements. The results clearly validate the proposed comparison method for calibrating the scale bar length. The technique is particularly useful when it is otherwise impossible to calibrate the scale bar with the LOS method.

5.3. Calibration result of vertical/horizontal three-sphere scale bar based on comparison method

We calibrated the three-sphere scale bar length using the comparison method. In this case, the three-sphere scale bar is the test bar, while we use another 2.3 m long nominal scale bar with 38.1 mm (1.5 in) SMR nests at both ends as the master bar. The purpose of this experiment is to determine the variation in the length of the three-sphere scale bar between the horizontal and vertical orientations as a result of gravitational loading. The results are shown in Table 3.

The laser tracker horizontal/vertical angles to the ends of the two bars were adjusted to within one degree when measuring their lengths, as discussed earlier. From Table 3, it can be seen that the length of the bar is 2299.650 mm in the horizontal orientation and 2299.634 mm in the vertical orientation. Thus, it is shorter in the vertical orientation by 16 μm when compared to the horizontal orientation, likely due to the influence of gravity and mounting conditions. We performed the experiments on three different days and noticed that the length in the vertical orientation was smaller than that in the horizontal orientation by 10 μm to 25 μm (not shown in table). The results indicate that the length of the scale bar we investigated changed by an amount not likely to be significant for TLS performance evaluation.

6. Uncertainty calculations

We discuss uncertainty budgets for the LOS, FOTF, and comparison methods in this section. For the purposes of this discussion, we will restrict the uncertainty calculations to the distance between the two spheres mounted on the ends of the three-sphere scale bar. Then, the definition of our measurand is the distance between the center of the two spheres (determined by a least-squares fit from sampled points on the surface [18]) at the ends of the scale bar.

Some sources of error affect the coordinate of a point measured in all three methods. We discuss these first.

(1) Laser tracker ranging contribution: For an absolute distance meter (ADM) laser tracker with a specification of 10 μm accuracy over the measurement range, assuming any value within that specification is equally probable, the uncertainty in the measurement of any target is $10/\sqrt{3}$ = 6 μm along the ranging direction. We assume that the specification provided by the manufacturer is valid over the temperature range of ±0.5 °C that we encounter in our laboratory and therefore we do not separately account for the effect of temperature on the refractive index of air and the resulting error in the range measurements of the laser tracker.
(2) Sphere concentricity with SMR: The concentricity of 14 spheres are measured on a CMM. The experimentally-measured standard deviation is 5 μm, and this value is considered as the standard uncertainty in the determination of the coordinate.
(3) SMR concentricity: The concentricity specification of the maximum distance between physical and optical centers of the SMRs is 2.5 μm. While this specification is over a spherical volume, for this discussion, we assume that the maximum eccentricity along each axis is also 2.5 μm. Assuming

Table 3
Calibration result of horizontal and vertical three-sphere scale bar length by the comparison method.

Posture	Bar name	Line-of-sight measurement (mm)		Direct measurement (mm)		Systematic error of length (mm)	Calibrated result by comparison method (mm)
		Mean	Std	Mean	Std		
Horizontal	Master bar	2300.054	0.002	2300.048	0.005	−0.006	/
	3-sphere bar	/	/	2299.644	0.005	/	2299.650
Vertical	Master bar	2300.050	0.001	2300.047	0.005	−0.003	/
	3-sphere bar	/	/	2299.631	0.004	/	2299.634

*The uncertainties of these results are given in Section 6.

this value as the upper bound with any value within that bound as equally probable (rectangular distribution), the standard uncertainty in the measured coordinate is 1.4 μm.

We next discuss the uncertainty budget for sphere center-to-center distance measurement in the three different methods.

6.1. Uncertainty budget for line-of-sight method

The LOS method involves measuring the center of the SMR mounted in each of the two extreme spheres of the three-sphere scale bar. The error sources associated with the laser tracker and the concentricity of the sphere and the SMR contribute to the uncertainty in the coordinate of each of the two measured centers. The concentricity between the optical and mechanical center is not relevant because that is a common mode error in the measurement assuming the same SMR is used for both nests. The center point coordinate uncertainty is then propagated to estimate the uncertainty in the point-to-point length and subsequently combined with temperature effects.

(1) Mechanical terms: Combining in quadrature the laser tracker ADM ranging contribution and the concentricity between the sphere and the SMR, we obtain a standard uncertainty of 8 μm for a single point coordinate. Therefore, the uncertainty in the length between the two extreme spheres is $8\sqrt{2}$ = 11 μm.

(2) Temperature contribution: Assuming ± 0.5 °C fluctuation of the temperature in the room as an upper bound with any value inside that bound as equally probable, and assuming thermal expansion coefficient of carbon fiber as 2×10^{-6}/°C, the standard uncertainty of a 2.3 m length is 1 μm. This term is included in the budget because the measurand is the length of the bar at the instant in time when it is measured by the TLS, which could well be several hours before or after calibration.

(3) Combined standard uncertainty: Combining the laser tracker contribution of 11 μm and the thermal contribution of 1 μm, the combined standard uncertainty in length is 11 μm. The expanded uncertainty (k = 2) is 22 μm.

6.2. Uncertainty budget for four orientation & two face method

The FOTF method involves the measurement of the center of the SMRs in the extreme spheres from four orientation of the laser tracker. Uncertainty calculations are shown here for the following dimensions: AC = 2.3 m and OB = 4 m, therefore OC = 4.2 m and θ = 16°, as shown in Fig. 4.

(1) Mechanical terms: The standard uncertainty along the ranging direction due to the laser tracker is 6 μm as described earlier. While some laser trackers have separate R_0 (zero error in the range) specification, we calibrated the R_0 parameter prior to our experiments and subsequently verified that the R_0 error was smaller than 5 μm. Assuming any value within that bound as equally probable, the standard uncertainty due to this term is 3 μm. Summing in quadrature, the standard uncertainty u_R along the ranging direction is 7 μm. In order to compute the uncertainty along the direction transverse to the laser beam, only the repeatability of the azimuthal axis is considered, not the overall accuracy of the axis. This is because all targets are measured in front- and back-face and averaged, and the length itself is the average from four horizontal orientations of the laser tracker separated by 90° each. This removes the influence of all non-ranging systematic sources of error in the laser tracker. Experimentally determined one standard deviation repeatability of the horizontal angle measurement is 0.1 millidegrees (based on 10 consecutive laser tracker measurements of point A in Fig. 4. The vertical angle repeatability does not affect this measurement because the reference length is oriented horizontally. This translates to a one standard uncertainty u_H of 7 μm (for OC = 4.2 m). The standard uncertainty in the determination of coordinates A and C along the direction of the length AC are $\sqrt{(u_R \sin\theta)^2 + (u_H \cos\theta)^2}$, or 7 μm, each. Summing this term in quadrature with the terms arising from the concentricity of the sphere with the SMR and the concentricity of the optical and mechanical center of the SMR, we obtain the standard uncertainty of 8.7 μm for a single coordinate. The standard uncertainty in the length is therefore $8.7\sqrt{2}$ = 12 μm.

(2) Thermal effects: Assuming ± 0.5 °C fluctuation of the temperature in the room as an upper bound with any value inside that bound as equally probable, and assuming thermal expansion coefficient of 2×10^{-6}/°C for carbon fiber, the standard uncertainty of a 2.3 m length is 1 μm.

(3) Combined standard uncertainty: Combining the mechanical contribution of 12 μm and the thermal contribution of 1 μm, the combined standard uncertainty in length is 12 μm. The expanded uncertainty (k = 2) is 24 μm.

6.3. Uncertainty budget for comparison method

The comparison method is a multi-step process. In the first step, we measure the lengths of the master bar and the test bar in the direct mode, noted as $L_{\mathrm{master,direct}}$ and $L_{\mathrm{test,direct}}$. And then, we obtain the master bar length using the line-of-sight method ($L_{\mathrm{master,LOS}}$). Then, we calculate the error e as $e = L_{\mathrm{master,direct}} - L_{\mathrm{master,LOS}}$. Finally, we correct the direct method length of the test bar to obtain the length had we measured it in the line-of-sight mode. Thus, $L_{\mathrm{test,LOS}} = L_{\mathrm{test,direct}} - e = L_{\mathrm{test,direct}} - L_{\mathrm{master,direct}} + L_{\mathrm{master,LOS}}$. We next discuss the uncertainty in each of the three terms above.

(1) The uncertainty in the master bar in the line-of-sight method is only due to the ranging errors in the laser tracker, thus it is $6\times\sqrt{2}$ = 8.5 μm. The master bar does not contain spheres, instead it simply has nests for 38.1 mm (1.5 in) SMRs. Therefore, concentricity between the center of the sphere and the SMR is not relevant. Also, as mentioned earlier, the concentricity between the optical and mechanical center is not relevant because that is a common mode error in the measurement assuming the same SMR is used for both nests.

(2) The uncertainty in the direct method of the master bar is only due to the repeatability in the range and angle measurements. The purpose of the master bar is to quantify the systematic errors in the laser tracker, and therefore, they are not included in the uncertainty calculations. Thus, the uncertainty in the measurement of a coordinate in the direct method is given by $\sqrt{(u_{\mathrm{R}}\sin\theta)^2 + (u_{\mathrm{H}}\cos\theta)^2}$ (see Fig. 4) where u_{R} and u_{H} are simply the repeatability terms, 1 μm and 7 μm, respectively. Thus, the uncertainty in length of the master bar measured in the direct method is $6.7\sqrt{2}$ = 10 μm.

(3) Thermal effects on the master bar: Temperature variation between the direct and LOS measurements of the master bar results in a change in the length of the scale bar. Assuming ± 0.5 °C fluctuation of the temperature in the room as an upper bound with any value inside that bound

as equally probable, and assuming thermal expansion coefficient of $2 \times 10^{-6}/°C$ for carbon fiber, the standard uncertainty of a 2.3 m length is 1 μm.

(4) Finally, we determine the uncertainty in the direct method measurement of the test bar. The repeatability of the laser tracker, the terms arising from the concentricity of the sphere with the SMR and the concentricity of the optical and mechanical center of the SMR, affect this measurement. Summing these in quadrature, we get the uncertainty of the length of the test bar measured in the direct method as 8.5 μm.

(5) Thermal effects on the test bar: Temperature variation between the time of calibration of the test bar and the time it is actually used for TLS testing will result in a change in the length of the test bar. As described in part 3 above, we assume that the standard uncertainty of a 2.3 m length is 1 μm due to this term.

(6) Combing the terms above, we obtain the uncertainty in the length of the test bar as 18 μm. The expanded uncertainty ($k = 2$) is 36 μm.

7. Conclusions

The three-sphere scale bar is one approach of realizing the tests described in the TLS performance evaluation standard ASTM E3125-17. In this paper, we have described two novel calibration methods for the three-sphere scale bar length using a laser tracker. One method is the four-orientation and two-face (FOTF) method for calibrating the length when the bar is in the horizontal orientation. The other method is the comparison method for calibrating the length of the bar in any orientation - horizontal, vertical or diagonal. We have outlined the theory behind the FOTF method, presented validation data for both methods, and experimental results on calibration of the three-sphere scale bar. The main conclusions are as follows:

(1) For the length calibration of the horizontal three-sphere scale bar, the FOTF method can remove systematic errors of the laser tracker. The resulting length is within a few micrometers of the length obtained using the line-of-sight (LOS) method. The FOTF method is more convenient to use, and is necessary when the line-of-sight measurement is not possible for a horizontal scale bar.

(2) The comparison method can be used for calibrating the length of a horizontal, vertical or diagonally oriented scale bar, assuming that the systematic errors of the laser tracker are the same when the range, horizontal, and vertical angles from the tracker to the measured points on the test bar and the master bar are the same. This method is useful when the LOS method of the laser tracker cannot be used for calibrating the bar length.

(3) The methods proposed here allow the calibration of the three-sphere scale bar in-situ, i.e., during or immediately preceding or after the realization of the ASTM E3125-17 test procedures. This reduces the chances of drift in the length of the bar and minimizes the uncertainty in the reference length during the testing process.

(4) We have presented uncertainty budgets for each of these methods. The $k = 2$ expanded uncertainty for the line-of-sight, four orientation & two-face method, and the comparison methods are 22 μm, 24 μm, and 36 μm respectively. The experiments show that the FTOF method and the comparison method result in scale lengths that are within a few micrometers of the LOS method. The uncertainties shown in this paper are an order of magnitude smaller than typical MPE values for TLS today, which can be several hundred micrometers to a few millimeters.

We have performed the complete set of test procedures described in the ASTM E3125-17 standard, many of which are realized using the three-sphere scale bar. We will discuss those results in a future publication.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors are grateful to Dr. Meghan Shilling and Geraldine S. Cheok for carefully reviewing this paper. The author Ling Wang would like to thank the support by the National Key Technologies Research & Development Program (2018YFB2101004), the Public Welfare Technology Application Research Project of Zhejiang Province Science and Technology Department (LGG18F030010), and the State Scholarship Fund of China Scholarship Council (No. 201808330612).

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